LOCATION OF LIQUID OVER THE CRUST OF AN EARTH-LIKE PLANET

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Abstract. Liquid is kept inside containers, as our common knowledge tells. However, the largest body of liquid on earth - the ocean of water - is just a "thin" layer of water blanketed over the outside of the roughly spherical crust of earth. Thus the question "is ocean secure from overflow?" is raised. Some basic analysis is presented with a hypothetical planet and a first order approximation of classical gravity.

Key words. Earth, ocean

AMS subject classifications.

Liquid does not have a fixed shape, so in our familiar daily life on the surface of earth, liquid stays "geometrically inside" some containers. However, this is not the case when the largest body of liquid on earth - the ocean of water - is examined.

The largest depth of the ocean on earth is about 11,000 meters [1], which is below 1/500 of the earth's radius 6371km [2]. Thus compared to the size of the earth, the vast ocean is only a thin layer of water blanketed over the outside of the roughly spherical surface of the earth. i.e. instead of staying inside some container, the ocean resides "geometrically outside" the ocean bed which is the solid surface supporting the ocean water.

To study the case of ocean, we extend the concept of containers to supports. A support has a solid surface in Euclidean space. For a given support surface S, a point x is geometrically inside S if there is a plane P, such that 1) point x is on the plane P, 2) the intersection of S and P is a loop, 3) on plane P, point x is inside the loop. 4) surface S and P collectively form a closed space.

There are 2 types of support. 1) liquid stays "geometrically inside" its support. e.g. water in a cup. 2) liquid resides "geometrically outside" its support. e.g. ocean water over the ocean bed.

In general, support and gravity collectively shape the body of liquid. For liquid geometrically inside a cup, the cup (the support) is small so the surrounding gravity can be considered uniform, the cup needs to have certain orientation with respect to surrounding gravity to keep the liquid from overflow.

But in the case of ocean over the earth, where the ocean bed is the support and water resides outside the support, the ocean bed is so large that the nonuniformity of gravity must be taken into account, the question "is ocean secure from overflow?" becomes complicated. Some related questions are: Can the ocean relocate? Why does the ocean reside where it is? These questions might be important, as an overflow of ocean would be a huge flood, and a relocation of ocean could easily submerge a continent.

The questions involve many factors, one of which is the spin of the earth. As earth's equatorial radius is slightly larger than its polar radius, if the earth spun slower, water on earth would be subject to weaker centrifugal force, thus amassing at the poles, then the poles would be submerged in the ocean and the crust along the equator would emerge as dry land. Otherwise, if the earth spun faster enough, centrifugal force

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would strengthen, thus water would accumulate at the equator, therefore land near the equator would be submerged in the ocean, and the crust at the poles would emerge as dry land.

Another factor is the density of mass. On earth, the density of ocean water is smaller than that of the materials underneath the crust, which allows the ocean water blanketing over the crust of the earth.

For simplicity, in our study of liquid over an earth-like planet, we assume 1) the planet does not spin. 2) the planet is terrestrial. 3) the density of liquid is smaller than that of the materials underneath the crust. 4) there is no other source of gravity.

Generally, the sea level is an equipotential gravitational surface, and 2 distinct equipotential gravitational surfaces do not have common points. If the crust of a planet is an equipotential gravitational surface, liquid over the planet will evenly cover the entire crust of the planet, leaving no dry land. Thus for dry land arising above sea level, the crust must be different enough from any equipotential gravitational surface. On the other hand, tectonic plates move, thus the shape of the crust changes over time. If the crust's shape approaches an equipotential gravitational surface, portion of crust that was dry land may be submerged in ocean.

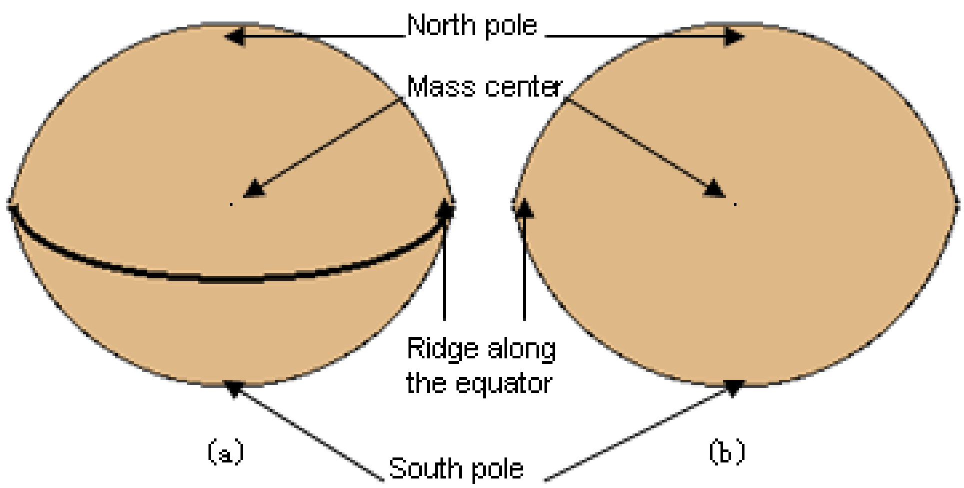

FIG. 0.1. *(a) crust of hypothetical planet X. (b) cross section along the axis*

To further study the case, we present a model of an isolated hypothetical planet X with a ridge along the equator, as shown in fig. 0.1. Planet X is a spatially extended object with distribution of mass $m(\mathbf{x})$, the surrounding gravitational potential is

$$V(\mathbf{x}) = -G \int \frac{m(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d\mathbf{x}' \tag{0.1}$$

the total mass of planet X is

$$M = \int m(\mathbf{x}) d\mathbf{x} \tag{0.2}$$

the mass center of planet X is at

$$\bar{\mathbf{x}} = \frac{\int \mathbf{x} m(\mathbf{x}) d\mathbf{x}}{\int m(\mathbf{x}) d\mathbf{x}} \tag{0.3}$$

Approximate $1/|\mathbf{x} - \mathbf{x}'|$ with first degree Taylor polynomial at $\mathbf{x} - \bar{\mathbf{x}}$

$$\begin{aligned} \frac{1}{|\mathbf{x}-\mathbf{x}'|} &= \frac{1}{|(\mathbf{x}-\bar{\mathbf{x}})-(\mathbf{x}'-\bar{\mathbf{x}})|} \\ &\approx \frac{1}{|\mathbf{x}-\bar{\mathbf{x}}|} + \frac{(\mathbf{x}-\bar{\mathbf{x}})\cdot(\mathbf{x}'-\bar{\mathbf{x}})}{|\mathbf{x}-\bar{\mathbf{x}}|^3} \end{aligned} \tag{0.4}$$

then the gravitational potential can be approximated around $\mathbf{x} - \bar{\mathbf{x}}$ as

$$\begin{aligned} V(\mathbf{x}) &\approx -G\int \frac{m(\mathbf{x}')}{|\mathbf{x}-\bar{\mathbf{x}}|}d\mathbf{x}' - G\int \frac{m(\mathbf{x}')(\mathbf{x}-\bar{\mathbf{x}})\cdot(\mathbf{x}'-\bar{\mathbf{x}})}{|\mathbf{x}-\bar{\mathbf{x}}|^3}d\mathbf{x}' \\ &= -G\frac{M}{|\mathbf{x}-\bar{\mathbf{x}}|} \end{aligned} \tag{0.5}$$

In this first order approximation, the gravitational potential $V(\mathbf{x})$ is calculated as if the total mass M locates at the center of the mass $\bar{\mathbf{x}}$. The first order approximation is used in our analysis due to the following reasons: 1) for a roughly spherical planet (e.g. our earth), the higher order terms are relatively small, 2) the calculation of higher order terms requires detailed distribution of mass of the planet, which may not be easily available.

From the first order approximation 0.5, the gravitational potential $V(\mathbf{x})$ is symmetrical with respect to the mass center, thus the center of mass $\bar{\mathbf{x}}$ is used as a reference point to compute the height of point $\mathbf{x}$ as $|\mathbf{x} - \bar{\mathbf{x}}|$, consequently the lower a point $\mathbf{x}$ is, the lower the gravitational potential $V(\mathbf{x})$ is. And water flows toward lower places.

On the crust of planet X shown in fig. 0.1, the ridge is the furthest from the mass center, thus the highest region, while the poles are the lowest regions. As a result, liquid on the crust of X flows toward the poles, away from the ridge, liquid on the north and/or south half of the crust tends to stay on its own half. Therefore the ridge tends to prevent water from flowing between the northern hemisphere and southern hemisphere. And liquid can cover only one half of the crust.

Fig. 0.2 illustrates planet X with its northern hemisphere being covered with ocean. Its mass center will shift northward accordingly, as eq. 0.3 implies. Then, the height of north crust is decreased, and the height of the south crust is increased. In another word, as liquid covers the northern hemisphere, the north half of crust (the ocean bed) becomes lower, and the southern hemisphere becomes higher.

With the configuration in fig. 0.2, the capacity of northern hemisphere is the maximal amount of liquid that can be supported on the northern hemisphere before liquid flows over the ridge onto the southern hemisphere. The capacity is determined by the ocean bed and the sea level. In the current setup, the ocean bed is the north crust. For water to remain over the northern hemisphere without crossing the ridge, the sea level must be lower than the ridge. Since a sea level is an equipotential gravitational surface, the highest sea level is the equipotential gravitational surface through the top of the ridge. As a result, the capacity of northern hemisphere is the volume defined by the north half of the crust and the equipotential gravitational surface through the top of the ridge.

The balance of the configuration in fig. 0.2 can alter due to movements of tectonic plates, because these movement changes the mass distribution of the planet, subsequently the mass center of planet X moves and equipotential gravitational surfaces shift. Since the mass center is the reference point, its drift can have several impacts.

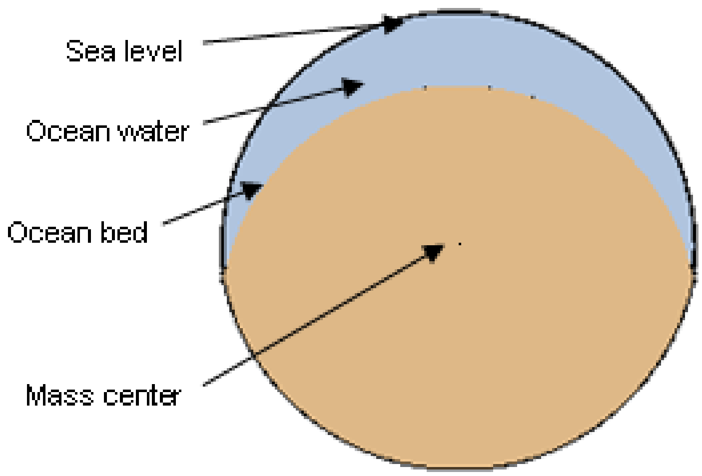

FIG. 0.2. *Cross section of planet X when water covers its northern hemisphere. The mass center shifts northward. The north half of the crust becomes lower, and the south half of the crust becomes higher. The sea level is an equipotential gravitational surface. Since the ridge is higher than the north pole, water tends to flow away from the ridge, thus the ridge tends to prevent water from flowing to the southern hemisphere.*

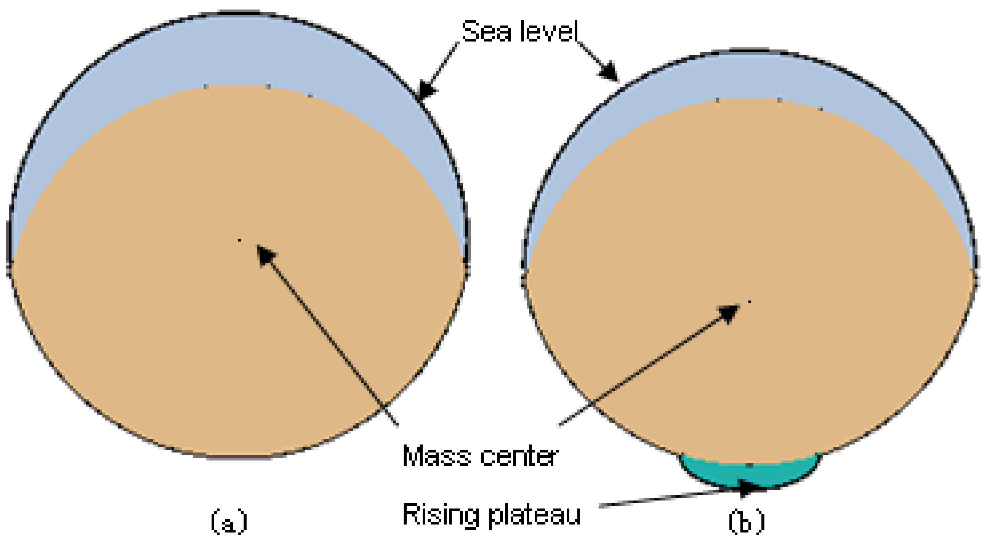

FIG. 0.3. *Cross section of planet X, a) water covers the north half of the crust. b) a plateau forms at the south pole, causing the mass center shifting southward. The equipotential gravitational surface through the top of the ridge becomes closer to the north half of the crust.*

In specific, if a plateau forms at the south pole, as shown in fig. 0.3(b), the mass center is shifted toward south pole, the height of the north half of the crust is increased, and the height of southern hemisphere excluding the plateau is decreased. In another word, ocean over the north half of the crust is pushed higher due to a plateau rising at the south pole.

Fig. 0.3 also shows that, mass center shift can change the capacity of the northern

hemisphere. As the mass center shifts southward, the equipotential gravitational surface through the top of the ridge becomes closer to the north crust, therefore the amount of liquid that can be supported on the north crust decreases, causing water flowing over the ridge into the southern hemisphere.

The overflow can cause the following impact chain, water flowing southward $\rightarrow$ mass center shifting southward $\rightarrow$ The north half of crust can support less water $\rightarrow$ more water flowing southward. The impact chain indicates a positive feedback.

Our model shows that rising plateau on one side of a planet can push the other side of the planet higher and decrease the capacity to store water, which may cause the ocean overflow. While in our presentation with a hypothetical planet, the ridge is along the equator, it may have various shapes. If our earth is considered for such analysis, one known huge rising plateau may be shifting the mass center of the earth by a notable amount. Though the first order approximation is used to illustrate our model, it is not enough for accurate calculation. For a planet as large as the earth, a small correction from the higher order terms may well shift the ocean level by a few meters, which is large enough to be felt by many people around the world. In addition, the spinning of the earth and the gravity of the moon also contribute to the location of ocean.

Acknowledgments.

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